A Monetary Model of Exchange Rate Beats the Random Walk Forecast Even at a Short Horizon: Evidence from the Serbian Hyperinflation at Daily Frequency

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Abstract: This paper finds that an out-of-sample forecast of a monetary model of exchange rate (MMER) in hyperinflation decisively beats a random walk one particularly at the most challenging one step ahead forecast, thus outperforming standard results previously obtained for low inflation episodes. The findings refer to the Serbian hyperinflation at daily frequency, and are robust with respect to various tests. Fast adjustment of the exchange rate to its fundamental value and the low, well below one discount factor found in the Serbian episode, as opposed to low-inflation ones, can account for divergent results in the respective inflation environments. The low discount factor appears in other hyperinflation episodes, while fast adjustment is due to the absence of nominal rigidities in hyperinflation thus both suggesting that reported findings for one episode might generalize.

Key words: Monetary model of exchange rate, Out-of-sample forecast performance, Random walk forecast, Hyperinflation at daily frequency.

JEL: F 31, F 47, E 31

1. *Introduction*

A long-lasting puzzle, put forward by Meese and Rogoff (1983a, 1983b and 1988), that fundamental based exchange rate models cannot outperform simple random walk forecast of no change, is still alive and well, as shown in a recent paper by Engel and Wu (2023). A prominence of this puzzle for standard open economy macro models is stressed by Itskhoki and Mukhin (2021) who placed it ahead of the other major puzzles in the field. The relevance of the issue of exchange rate predictability is forcefully put forward by Rogoff (2007) stating that the aim is to find models that can forecast exchange rate variability and hence can be used for policy analysis.

This paper addresses this continuing puzzle of, particularly challenging, short-horizon exchange rate forecasts while examining the predictive power of a monetary model of exchange rate (MMER) in hyperinflation at daily frequency. Specifically, the paper aims to show that an MMER can outpredict a random walk (RW) model better for a hyperinflation episode than for a low-inflation one. Although it has already been demonstrated that the MMER fares better in hyperinflation (see Engsted, 1996, and Petrovic and Mladenovic, 2000), its ability to predict has not been explored due to a severe shortage of monthly observations. An abundance of daily data for a specific episode, the Serbian hyperinflation, now removes this constraint. Moreover, very high, daily frequency and extremely short, one-day-ahead forecasts pose a tremendous challenge to an MMER even in hyperinflation. Finally, we explore whether the results obtained for a single hyperinflation episode could be generalized.

The Serbian hyperinflation, that we use as an “experiment” to assess the predictive power of an MMER, erupted in 1992. and lasted through January 1994., that is at the beginning of the Serbian transition towards full-fledged market economy. An interplay of transition process in Serbia and disintegration of Yugoslavia, with subsequent war conflicts, led to hyperinflation outburst. (For the main facts on the Serbian hyperinflation and stabilization see Bogetić et al., 2022, and Mladenović and Petrović (2010). For interaction of inflation and exchange rate depreciation in hyperinflation and in its aftermath see Petrović et al., 1999., and Petrović and Mladenović, 2015). In that respect, Serbia joined a number of Central and Eastern European (CEE) countries, specifically Poland (1989 -1990), Bulgaria (1996 - 97), Ukraine (1992 - 94) and Russia (1992-93), that also experienced hyperinflation at early stages of its transition to market economy.

Other transition CEE countries, at the beginning, also underwent spells of high inflation as they liberalized its prices and exchange rates, while introducing currency convertibility, etc. but quickly recourse to macroeconomic stabilization to tame initial imbalances, now in a new decentralized, market environment. This early stage of transition is thoroughly studied in Blanchard (1997) and Blejer and Škreb (1997) among others, and they give a context for the Serbian hyperinflation and shot-lived stabilization episode. Thus, Blanchard (1997), upon offering main stylized facts across countries, explored mechanism of the transition process. In Blejer and Škreb (1997) collection of papers, beside the study of general transition patterns, country studies are presented, the relevant for this paper being those on inflation and stabilization in Poland, Slovenia, Hungary, Croatia and the Baltic states.

In contrast, Serbia pursed short-lived stabilization, that halted hyperinflation, but in the absence of structural reforms, failed to contain inflation and related currency depreciation (Bogetić et al, 2022, and Petrović and Mladenović, 2016).

The paper proceeds as follows. We start with literature review given in section 2. Section 3 presents an MMER to be used for forecasting in hyperinflation, motivates it with a fundamental-based exchange rate model, and compares the model to a standard one. A background of the Serbian hyperinflation and characteristics of the series are examined and tested in section 4. Section 5 inspects whether an MMER outpredicts a random work model by testing their respective out-of-sample forecasting power. Section 6 offers an explanation for good MMER forecast in the Serbian episode and examines whether this result might generalize. Section 7 concludes.

2. *Literature review*

Meese and Rogoff (1983a, 1983b and 1988) in their seminal papers advanced a puzzle that exchange rate fluctuations are unpredictable, implying its disconnection with economic fundamentals. More specifically they suggest that no change random walk exchange rate prediction is superior to the one base on economic models.

Extensive research on whether exchange rate is predictable ensued, and thorough assessment in subsequent surveys by Frankel and Rose (1995), Engel, Mark, and West (2007), and (Rossi, 2013), concluded: "it depends" (Rossi, 2013). Namely, depending on economic model used, forecasting horizon, econometric specification, currency combinations and etc. (see Rossi, 2013, and Cheung et al., 2005, 2019), it is found that the puzzle holds or not.

Econometric models, used to forecast exchange rates, could be grouped into three categories: single-equation, multiple equations and panel models (Rossi (2013). Each category was further clustered allowing for either linear or nonlinear specification, cointegration being present or not, and linear or time-varying parameters. Single-equation models were found to be more successful, among them particularly ECM, at the long-run forecasting horizon. Within ECM specification the forecasting ability was also associated with the approaches used to obtain cointegration parameters.

Concerning fundamental based exchange rate models, it is obtained that monetary model of exchange rate, where money supply is the main predictor, forecasts well exchange rate over long-run (more than years) in a single country (Mark, 1995). Same result is also found in a panel of countries, that is monetary model again predicts well exchange rate over long horizon (see Mark and Sul 2001, Groen 2005, Engel, Mark, and West 2007, and Cerra and Saxena 2010). In both cases linear, error-correction model (ECM) is used, and the known cointegrating coefficient is included in the ECM.

Nevertheless, over short horizon (less than 2–3 years, Rossi, 2013) it is found that monetary model performs poorly in predicting exchange rate fluctuations (i.e., Cheung, Chinn, and Pascual 2005). In contrast, there is some evidence that exchange rate can be forecasted well even in the short-run, using alternative models. Thus, Molodtsova and Papell (2009) used Taylor-rule fundamentals as predictors and obtained good forecasts over short horizon. Gourinchas and Rey, (2007) found that net foreign assets predict well exchange rate fluctuation both in short and long-run. However, Rogoff and Stavrakeva (2008) show that the reported progress in forecasting exchange rate is still modest at short horizons.

Engel and Wu (2023) in their topical paper questions even the findings that models, advanced in the recent literature, predict well exchange rates over medium- and long-run. They point out that tests used to asses predictability have small-sample bias, and upon addressing this issue they, sadly, found that the random walk hypothesis cannot be rejected.

Against the backdrop of the reviewed literature, we use monetary model of exchange rate in hyperinflation, with money supply as the sole predictor, to forecast over short horizons. While doing so, we estimated ECM for the single country and imposed a cointegrating parameter estimated in advance. We show, in contrast to previous studies, that this model forecasts well exchange rate even in the short-run.

While concluding this brief literature survey, let us also mention two recent unorthodox lines of research. The first one combines, previously reviewed, fundamental based models of exchange rate, with machine learning (Amat et al. 2018, and Zhang and Hamori 2020), suggesting that they outcompete the predictability of random walk. The second line explores whether cryptocurrency returns are predictable using random walk as a benchmark, that is examining if the Meese–Rogoff puzzle holds in this case. This is natural extension since exchange rate can be treated as an asset price, but the findings suggesting predictability are still very preliminary (Magner and Hardy, 2022).

3. *An Exchange rate forecasting model in hyperinflation: An MMER*

The out-of-sample evaluation of exchange rate models is, since Mark (1995), generally based on an error-correction model (ECM) where the nominal exchange rate adjusts to its fundamental value. (Cf. Rogoff and Stavrakeva, 2008, and Molodtsova and Papell, 2012). The corresponding error-correction model exists if the exchange rate is cointegrated with its fundamental value.

In the case of a monetary model in hyperinflation, domestic money supply is the only macroeconomic fundamental that determines the long-run equilibrium level of the nominal exchange rate, as money growth exceeds by far that of all other fundamentals. Moreover, in hyperinflation and in a number of high inflation episodes, (logarithms of) the exchange rate (e) and the fundamental, money supply (m), are typically non-stationary I(2) processes (cf. Engsted, 1996, Petrovic and Mladenovic, 2000, and Phylaktis and Taylor, 1993), and therefore only their first differences (Δe and Δm) are I(1). Consequently, one should look for cointegration between Δe and Δm and, if present, use the corresponding ECM, i.e.:

$$\Delta^2 e_t = \gamma(\Delta m - \Delta e)_{t-1} + \text{short-term dynamics} + \varepsilon_t \qquad (1)$$

to assess the out-of-sample forecast of a monetary model. Since short-term dynamics in (1) include the lagged depreciation changes ($\Delta^2 e_{t-1}, \ldots$) the latter should also be included in a RW model (2) so that superior forecast of MMER (1) could be attributed solely to monetary variable.

Specifically, the forecast of the change in return ($\Delta^2 e$) from the model (1) will be confronted with that of an extended RW model that includes the lagged depreciation changes (of order p):

$$\Delta^2 e_t = \delta_1 \Delta^2 e_{t-1} + \delta_2 \Delta^2 e_{t-2} + \ldots + \delta_p \Delta^2 e_{t-p} + \varepsilon_t \qquad (2)$$

Comparison of the mean squared forecasting errors (MSFE) from two nested models above would then answer our question about whether a monetary model is able to outpredict a naïve RW forecast.

The standard approach, however, forecasts the exchange rate return (i.e. Δe), and not its change ($\Delta^2 e$), as the former is generally stationary apart from high inflation episodes. Thus, the same two nested models as (1) and (2) above are used except that Δe replaces $\Delta^2 e$, and it adjusts to the deviation in the levels of the exchange rate and its fundamental value (cf. Rogoff and Stavrakeva, 2008, p. 9, and Molodtsova and Papell, 2012, p.10).

Nevertheless, to make our results comparable to those in previous studies, we can obtain from a (1) MMER forecast of the exchange rate return as:

$$\Delta e_{t+1}{}^f = \Delta e_t + \gamma(\Delta m - \Delta e)_t + \text{short-term dynamics}$$

or

$$\Delta e_{t+1}{}^f = (1-\gamma)\Delta e_t + \gamma \Delta m_t + \text{short-term dynamics} \qquad (1a)$$

and from (2) that of an extended RW model:

$$\Delta e_{t+1}{}^f = \Delta e_t + \delta_1 \Delta^2 e_{t-1} + \delta_2 \Delta^2 e_{t-2} + \ldots + \delta_p \Delta^2 e_{t-p} \qquad (2a)$$

Since the respective forecasting errors do not alter as one switches from predicting the change in return (1 and 2) to predicting the return itself (1a and 2a), cf. Appendix, one may stick to (1) and (2) while confronting the mean squared forecasting errors (MSFE) from the two models, and testing whether MMER forecast can outperform an RW one. Additional intuition is gained from expressions 1a and 2a, showing that, short-run dynamics aside, MMER predicts the future exchange rate ($\Delta e_{t+1}{}^f$) in the case of fast adjustment i.e. γ=1,as equal to the current money supply (Δm_t) contrary to an RW model that forecasts it with the current exchange rate (Δe_t). The difference between the two forecasts diminishes with γ i.e., when the speed of adjustment of the exchange rate to its fundamental decreases.

A motivation for an MMER (eq. 1 above), could come from a fundamental-based exchange rate model stating that the exchange rate is determined by a discounted present value of future fundamentals:

$$e_t = (1 - a)\, \Sigma_{i=0}^{\infty}\, a^i E_t\, m_{t+i} \qquad (3)$$

where $a = \alpha/(1+\alpha)$ is the discount factor, and α semi-elasticity of money demand. From (3) one can derive (cf. Engsted, 1993):

$$e_t = m_t + \Sigma_{i=1}^{\infty} a^i E_t \Delta m_{t+i} \qquad (4)$$

and

$$e_t = m_t + \alpha \Delta m_t + (1- a)^{-1} \Sigma_{i=1}^{\infty} a^i E_t \Delta^2 m_{t+i} \qquad (5)$$

As explained above, the exchange rate and money supply are I(2) processes in hyperinflation, hence one may ignore the last term in (5) being stationary, or alternatively one may follow Mark (1995) assuming, in the current context, that money growth (Δm_{t+i}) follows a driftless random walk and substitute it in (4). In either case one gets:

$$e_t = m_t + \alpha \Delta m_t \qquad (6)$$

which somewhat differs from the corresponding standard low-inflation case: $e_t = m_t$ (e.g. Mark, 1995) when variables are I(1) processes. Nevertheless, even in (6) m_t being I(2) process overwhelms its first difference (Δm_t) in determination of e_t which itself is I(2) process.

Finally, one may differentiate (6):

$$\Delta e_t = \Delta m_t + \alpha \Delta^2 m_t \qquad (7)$$

and explore whether I(1) variables Δe_t and Δm_t cointegrate, and hence the corresponding ECM exists where the exchange rate adjusts to its fundamental value:

$$\Delta^2 e_t = \gamma(\Delta m - \Delta e)_{t-1} + \gamma \alpha \Delta^2 m_{t-1} + \varepsilon_t \qquad (1b)$$

The expression (1b) is encompassed by MMER (1) above, and the last term: $\gamma \alpha \Delta^2 m_{t-1}$ might motivate the inclusion of the short-term dynamics in (1).

4. *The Serbian hyperinflation: Stylized facts and testing*

The episode to be examined covers the most severe portion of the extreme Serbian hyperinflation of 1992-1993 at daily frequency, i.e. from July 1st to December 10th. For in depth analysis of the Serbian hyperinflation and stabilization see Bogetić et al. (2022); for monetary dynamics Mladenović and Petrović (2010), and for interaction of inflation and exchange rate depreciation in hyperinflation and in its aftermath see Petrović et al. (1999), and Petrović and Mladenović (2015).

Although hyperinflation ran through January 1994, the sample is shorten as some observation on money supply are missing. They were interpolated for some of the analyses given in above mention papers. However, using interpolated data might affect forecasting tests' results, hence the shortening of the sample used.

The severity of the portion we are looking at is illustrated by the average (see Table 1) and actual (Figures 1 and 2) daily rates of exchange rate depreciation and money growth, as well as their standard deviations.

Table 1

Exchange rate Depreciation (Δe) and Money Growth (Δm)

Per day (%)

	July	August	September	October	November
Δe					
Mean	9.2	7.7	7.2	11.6	21.5
St. dev.	9.5	7.9	8.7	13.5	28.3
Δm					
Mean	8.1	7.6	6.7	10.1	18.0
St. dev.	8.7	6.7	6.4	14.5	14.9

Note: The rates above are defined as log difference, e.g. $\Delta e = \ln(E/E_{-1})$.

Figure 1

Exchange Rate Depreciation (Δe)

Daily frequency

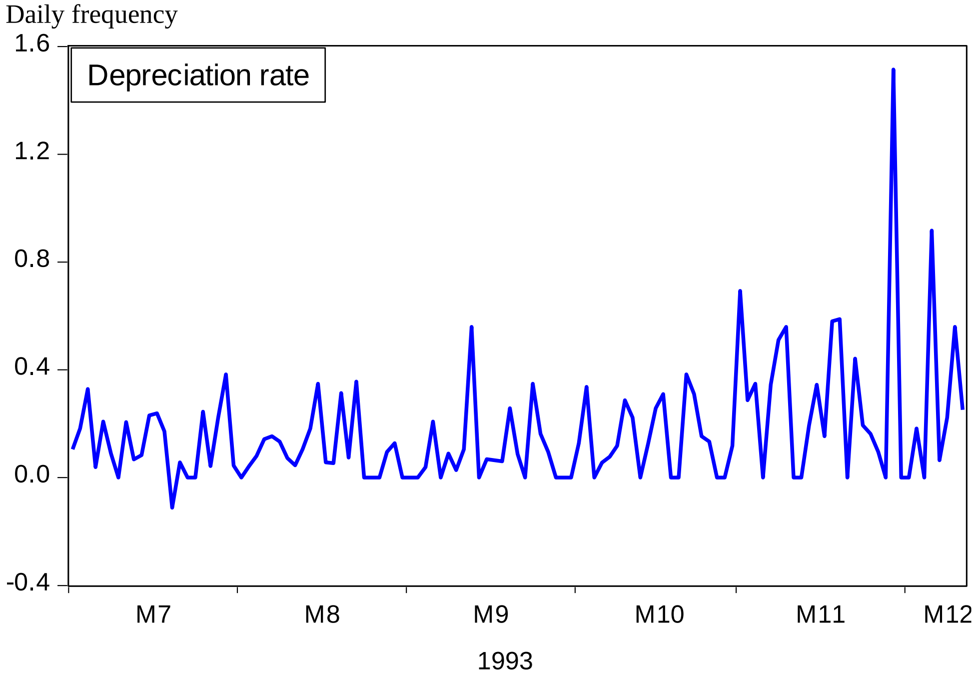
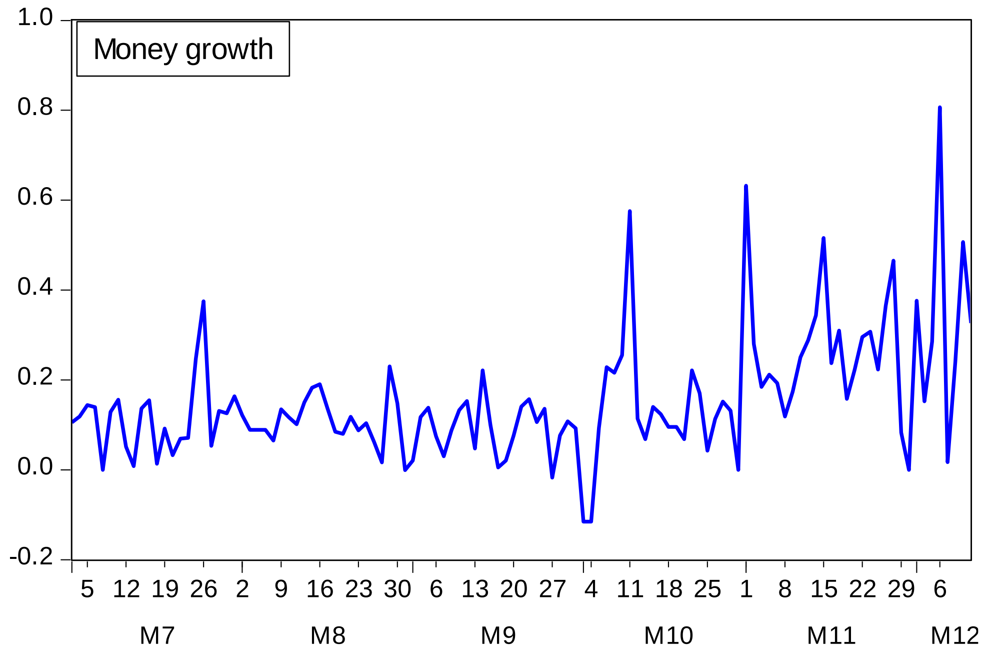


Figure 2

Money Growth (Δm)
Daily frequency



The data to be used are black-market exchange rates and the currency in circulation (cash) as money supply, both with a daily frequency. These series are relatively sound compared to other data in hyperinflation. Data for exchange rates were generated by the black market and recorded daily in newspapers, as practically all transactions were carried out or quoted in foreign currency, specifically in German marks. This is particularly true for the most severe portion of hyperinflation we are looking at. The source for the daily cash series is the central bank of Serbia. Again this series is relatively sound. Namely, the central bank, as the printer and distributor of cash, had direct control and evidence of cash expansion, and thus was able to record its magnitude quite accurately. All reported estimates are based on samples that cover five working days per week, since the money supply did not change over weekends.

Expressions presented in section 3 imply certain characteristics of the variables and relations between them, and we shall now test whether they hold in the Serbian episode.
As suggested, unit root testing does confirm that both exchange rate (e) and money supply (m) are I(2) processes and that they cointegrate such that (m - e) is an I(1) process (see Table 2). The latter validates eq. 4 above.

Table 2
Unit Root Testing
(Period: July 1 – December 10, 1993)

Augmented Dickey-Fuller (ADF)			
	e_t	m_t	m_t-e_t
H_o:I(3) H_1:I(2)	-17.99	-15.98	
H_o:I(2) H_1:I(1)	-2.07	-1.66	-11.47
H_o:I(1) H_1:I(0)			-1.94
Kwiatkowski-Phillips-Schmidt-Shin (KPSS)			
	e_t	m_t	m_t-e_t
H_o:I(2) H_1:I(3)	0.004	0.005	
H_o:I(1) H_1:I(2)	0.21	0.23	0.012
H_o:I(0) H_1:I(1)			0.29
Elliott-Rothenberg-Stock (ERS)			
	e_t	m_t	m_t-e_t
H_o:I(3) H_1:I(2)	-11.52	-15.65	
H_o:I(2) H_1:I(1)	-1.01	-1.47	-11.47
H_o:I(1) H_1:I(0)			-2.00

Note: The number of lags is chosen according to modified Schwarz criterion. The number of corrections is equal to 11 for exchange rate and money, and 6 for real money, in the first phase of testing, and 0 in the second phase of testing for all time series. The unit root tests are based on the model with constant and trend with the 5% critical value for ADF –3.45 (MacKinnon, 1991) and -3.03 for ERS (Elliott, Rothenberg and Stock, 1996). The corresponding 5% critical value for the KPSS test is 0.15 (Kwiatkowski, Phillips, Schmidt and Shin, 1992). The 5% critical value for the right tail of the ADF distribution is –0.90 in the model with constant and trend (Fuller, 1976).

As I(2) variables *e* and *m* cointegrate making (*e*-*m*) I(1) process (see Table 2), it follows that $\Delta(m - e)$ is stationary, and since Δe and Δm also cointegrate (see Table 3), the ECM (1) above exists.

Table 3

The Johansen cointegration test between Δe and Δm

(Period: July 1–December 10, 1993)

Rank	Eigenvalue	Trace test	Cointegrating vector	
r=0	0.285	39.49	Δe_t	Δm_t
r≤1	0.002	0.27	1	-1.070

Note: There is no constant in the VAR model. There are six lags in the VAR model. The following impulse dummy variables are included in the VAR to model outliers that are identified as extreme values of standardized VAR residuals. These are: D1, D2 and D3. The dummy variables are defined as follows: D1 = 1 for 1993:11:1 and 0 otherwise, D2 = 1 for 1993:11:9 and 0 otherwise, and D3 = 1 for 1993:11:29, -1 for 1993:11:30 and 0 otherwise. The 5% bootstrapped critical values for the trace test, obtained within Oxmetrics 8, are: 22.10 for r=0 and 9.05 for r≤1, so that corresponding p-values are: 0.00 and 0.66 respectively.

Finally, (*e*-*m*) cointegrates with Δm_t supporting eqs. 5 and 6 above, and giving an estimate of money demand semi-elasticity equal to $\alpha = 5.37$ (see Table 6 and Figure 5 in Mladenović and Petrović, 2010).

Thus, for forecasting exchange rate we shall use version of MMER (1b), over one, three - and five days horizon. The corresponding estimates of the model are presented in Table 4.

Table 4

The MMER estimated models used for forecasting
(Period: July 1– September 1, 1993)

For *k=1*, Dependent variable: $\Delta e_{t+1} - \Delta e_t$

Regressor	Estimate	t-ratio
$(\Delta m_t - \Delta e_t)$	1.03	6.69
$(\Delta m_t - \Delta m_{t-1})$	-0.62	-2.89
$(\Delta m_{t-1} - \Delta m_{t-2})$	-0.53	-2.58

Adjusted R^2=0.50, AR 1-7=1.14(0.36), JB=0.92(0.63), ARCH(1)=1.27(0.27), HETERO=1.12(0.36), RESET=0.57(0.57)

For *k=3*, Dependent variable: $\Delta e_{t+3} - \Delta e_t$

Regressor	Estimate	t-ratio
$(\Delta m_t - \Delta e_t)$	0.88	5.77

Adjusted R^2=0.43, AR 1-7=0.85(0.56), JB=1.42(0.49), ARCH(1)=1.40(0.24), HETERO=0.57(0.57), RESET=0.45(0.64)

For k=5, Dependent variable: $\Delta e_{t+5} - \Delta e_t$

Regressor	Estimate	t-ratio
$(\Delta m_t - \Delta e_t)$	0.41	2.15
$(\Delta e_t - \Delta e_{t-5})$	-0.39	-2.38

Adjusted R^2=0.38, AR 1-7=0.93(0.49), JB=1.63(0.44), ARCH(1)=0.03(0.86), HETERO=0.17(0.95), RESET=0.15(0.85)

Note: AR 1-7 is F test-statistic for up to seventh-order serial correlation in the residuals; JB is the Jarque-Bera test statistic for normality of the residuals with chi2(2) distribution under the null; ARCH(1) is the F statistic for testing first-order autocorrelated squared residuals; RESET is the regression specification F test that tests the null of correct specification against the alternative that residuals are correlated with squared and cubed fitted values of the explanatory variables; HETERO is the White residual heteroskedasticity test without cross-terms that is also given in the form of F test.

5. *Can an MMER outpredict the random walk?: Testing the out-of-sample forecasting power of an MMER vs. the random walk*

While assessing an MMER out-of-sample forecast we shall focus on the short horizon, i.e. one-period-ahead forecast, since at this short horizon, fundamental-based exchange rate models fare poorly. It is only with the long-horizon, i.e. several-periods-ahead forecast, that some encouraging results are found (cf. Engel, Mark and West, 2007). Nevertheless, we shall also evaluate a several-steps-ahead forecast.

An MMER out-of-sample prediction is obtained by splitting the sample into two portions, an in-sample and out-of-sample one, using the former to estimate the ECM (eq. 1) and then to make the forecast. The standard procedure is that for each new forecast an additional observation is added to the in-sample portion and the parameters of the model re-estimated. This can be done using either a recursive or rolling specification (cf. Engel, Mark and West, 2007, and Rogoff and Stavrakeva, 2008), and we opted for the former. The recursive method simply adds new observations thus increasing the in-sample portion used for estimation, while rolling specification preserves in-sample portion constant by adding new and dropping initial observations.

Thus, the ECM (eq. 1) is initially estimated for the in-sample portion running from July 1st to September 1st, and then employed to forecast $\Delta^2 e$ one day ahead (t+1), using information available in the current period (t). The previous procedure is repeated by adding an additional observation to the sample, estimating the model, and forecasting. In a similar manner, a k-steps ahead forecast is obtained again using information available in the current period (t) and the following ECM (cf. Mark,1995) that includes short-term dynamics:

$$\Delta e_{t+k} - \Delta e_t = \gamma_k (\Delta m_t - \Delta e_t) + \beta_{1k} (\Delta e_t - \Delta e_{t-k}) + \beta_{2k} (\Delta m_t - \Delta m_{t-k}) + \ldots + \varepsilon_{t+k,t} \qquad (8)$$

Thus expression (8), as opposed to (1), relates multiple-period (k) changes ($\Delta e_{t+k} - \Delta e_t$) to the initial deviation of the exchange rate from its fundamental value, asking whether long-horizon exchange rate changes are predictable. The motivation is that, while short-horizon changes may be dominated by noise, this noise might average out over time thus revealing a systematic relationship such as (8).

The same procedure is used to generate forecasts from the extended RW model: one-step ahead from (2), and k-step ahead from:

$$\Delta e_{t+k} - \Delta e_t = \delta_{1k}(\Delta e_t - \Delta e_{t-k}) + \delta_{2k}(\Delta e_{t-1} - \Delta e_{t-k+1}) + \ldots + \varepsilon_t \qquad (9)$$

Testing whether an MMER can outpredict an RW model is based on comparing respective out-of-sample forecasting errors. Specifically, one wants to test the null hypothesis that the respective mean squared forecasting errors (MSFE) are equal against the alternative hypothesis that the MSFE of the RW model (2 and 9) is larger than the MSFE of the MMER (1 and 8 respectively), i.e. that their difference is significantly positive. This is exactly what the Diebold and Mariano (1995), and the West (1996) asymptotic test (DMW) examines. The problem with the asymptotic DMW test is that in a small sample it is biased towards the null, stating that the structural model cannot outperform an RW forecast. As a way out, Clark and West (2006 and 2007) advanced test statistic (CW) that adjusts the DMW one, while correcting its size distortion. However, an issue with the CW test is that it is a nested test that under the null states that a RW model (eq.2) is the true model against the alternative that the structural model (eq.1) is a correct one. Therefore, even if the CW test rejects the null that the true model is a random walk, it does not necessarily follow that the MSFE from a RW model is larger than that from the structural one, and that the CW statistic is not a minimum mean-squared-forecasting-error (MSFE) statistic (cf. Rogoff and Stavrakeva, 2008, and Molodtsova and Papell, 2012, pp.11 and 12). Finally, the Theil-U statistics is also used, being the ratio of the MSFE of the MMER to that of the RW model, which is equal to 1 under the null and less than 1 under the alternative hypothesis. Therefore, we shall apply the bootstrapped version of all three statistics while testing whether MMER can out-predict a RW model.

Thanks to abundant daily data set, we used very large forecast window for hyperinflation episode consisting of 72 daily observations (see Table 5), and it is 1.6 times larger than the (initial) in-sample portion (45 observations). *Forecast window* is defined as the part of the sample for which forecasts are calculated (cf. Rogoff and Stavrakeva, 2008). Thus e.g. the September 2nd – December 10th (72 days) forecast window means that the first forecast is based on the in-sample portion running from July 1st to September 1st, i.e. 45 days.
Namely, data sets for hyperinflations are, almost exclusively, available only at monthly frequency and contain up to 30 observations, making it impossible to test the MMER out-of-sample forecast power. This ratio (1.6) of the post-sample observations - forecast window (P) to the in-sample ones (R), is somewhat lower than the one used in other studies, i.e. 2.5 in Engel, Mark and West (2007) and 2.7 in Moldtsova and Papell (2009), albeit for low inflation cases with plentiful data sets.

Attained out-of-sample forecasts of an MMER and an extended RW are confronted and testing results are reported in Table 5.

Table 5

Testing the out-sample-forecast performance of MMER vs. RW for various forecast horizons

Forecast window 72 days (September 2nd – December 10th, 1993): P/R = 1.6

k-days-ahead-forecast	Theil-U statistic	DMW statistic	CW statistic	Adjustment coefficient γ_k	Adjusted R_k^2
One-day-ahead	0.74(0.00)	2.13(0.02)	3.24(0.00)	1.03 (0.00)	0.50
Three-days-ahead	0.72(0.00)	1.50 (0.07)	2.72 (0.00)	0.88 (0.00)	0.43
Five-days-ahead	0.92(0.04)	0.35 (0.37)	2.13 (0.02)	0.41 (0.00)	0.38

Note: All three tests are one-sided where the null hypothesis that the two models have equal predictability power is tested against the alternative that an MMER outperforms a RW. The p-values are reported in parentheses, and they are the bootstrapped version of the corresponding test statistics. The bootstrap is based on 10000 replications. P/R is the ratio of the post-sample observations forecast window (P) to the in-sample ones (R).

The results presented in Table 5 clearly show that the MMER out-of-sample forecast outperforms the RW one at the short one-day-ahead horizon. Specifically, all three tests reject the null hypothesis that the two models have the same forecasting power in favor of the alternative that an MMER beats an RW forecast. Similarly, the MMER out-predicts the RW model for three and five-days-ahead forecasts. The Theil-U and the CW statistics clearly suggest that, while the DMW statistic questions the superiority of the MMER in the case of the five-day-ahead forecast (see Table 5).

The findings reported in Table 5 do suggest that forecasting power of the MMER somewhat decreases with increase in forecast horizon (k). This pattern contradicts the standard finding (cf. Mark, 1995, and Engel, Mark and West, 2007) that fundamental-based exchange rate models forecast poorly at one step ahead, only to improve their performance substantially for several-steps-ahead forecast.

It is the speed of adjustment of the exchange rate to its fundamental value that drives the difference between our finding and previous ones. The speed of adjustment is captured by the adjustment coefficient γ_k (in eqs. 8 and 1) and we found it to be 1 in one-day-ahead forecasts, and subsequently decreasing (cf. Table 5). This implies that the exchange rate (Δe) almost fully (apart for impacts of the short-run dynamics, cf. eqs. 1 and 8) adjusts to its fundamental value (Δm) in one period of time (k=1). Therefore, extending the forecast horizon several steps ahead (k >1) will only worsen the forecast, as results in Table 5 confirm. Same as the slope γ_k, R_k^2 also declines as the forecast horizon (k) increases (see Table 5) showing that the corresponding ECMs (8) is losing its predictive power. Engel, Mark and West (2008), pp. 38-39, show that R_k^2 may take on a humped-shape pattern in k.

The opposite pattern is found in previous studies where γ_1 is well below 1, and increases with k, together with $R_k{}^2$ until both reach respective maximum values (see Mark, 1995). The former result suggests that the exchange rate adjusts only partly to its fundamental value in one period of time (k=1), and hence keeps on adjusting over several (k >1) periods in the future. Consequently, in this case it is found that a several-steps-ahead forecast is better than a one-step-ahead forecast (Mark, 1995, and Engel, Mark and West, 2007).

6. *Why are good forecasts obtained?: Generalizing results for hyperinflation*

The found superiority of an MMER forecast in hyperinflation can be attributed to the high speed of adjustment of the exchange rate to its fundamental value and the low, well-below-one discount factor both detected in the Serbian episode as opposed to low-inflation ones.

As reported above, the adjustment coefficient for a one-step-ahead forecast in the Serbian hyperinflation is close to one, while the corresponding estimates for low-inflation episodes vary from 0.035 to 0.074 (cf. Mark, 1995, Table 2: US dollar against a four leading currencies). Moreover, in these low-inflation episodes, the adjustment coefficient becomes one only for 12- to 16-steps-ahead forecasts, indicating slow adjustment of the exchange rate to changes in economic fundamentals. The latter could be attributed to the presence of nominal rigidities (cf. Mark, 1995, and the corresponding references). However, in hyperinflation, variables including the exchange rate become extremely flexible, thus erasing nominal rigidities. This can then explain the sharply differing results found in the Serbian episode and in the standard low-inflation ones, but also tentatively generalize findings for a given episode to hyperinflation in general.

A low, well-below-one discount factor strongly increases the predictive power of a fundamental-based exchange rate model such as (3), by giving greater weight to current fundamental, money supply (m_t), relative to future ones, hence enhancing the power of the former (m_t) to predict the exchange rate (e_t). This can then explain the strong predictive power of the MMER above (eq. 1) in the Serbian episode.

Namely, as reported in section 4, semi-elasticity in the Cagan money demand (α) in the Serbian episode is estimated to be 5.37, giving a discount factor of 0.84. The latter is well below 1 and significantly lower than the 0.9 that Engel and West (2005) take as a benchmark value. Namely, for the discount factor to be 0.9, money demand semi-elasticity should be 9, and this is significantly higher than the obtained estimate: 5.37 (chisquared(1)=8.39(0.00)).

Again, contrary to the Serbian episode, empirical evidence from developed, low-inflation countries do suggest that the discount factor is close to unity. Thus, for the four main world currencies against the US dollar, Sarno and Sojli (2009) found, using monthly data, that the discount factor varies between 0.985 and 0.993, with an average value of 0.989. Nevertheless, as required by the model (3) above, all discount factor coefficients are significantly less than one. A large discount factor in low-inflation episodes can account for the observed feeble link between the exchange rate and economic fundamentals, and consequently for the weak predictive power of a fundamental-based exchange rate model (cf. Engel and West, 2005).

The low discount factor found in the Serbian episode, however, also appears in other hyperinflation episodes (see Table 6) thus suggesting that the findings in this paper may generalize. Reported discount factors are calculated using semi-elasticity estimates of the Cagan money demand.

Table 6

Semi-elasticity of Money Demand and the Discount Factor in Hyperinflation
Monthly frequency

Hyperinflation episodes

	Semi-elasticity	Discount factor $a = \alpha/(1+ \alpha)$	Average monthly inflation rate
Austria	3.8	0.79	38.5%
Germany	5.3	0.84	144
Hungary	8.3	0.89	37.8
Poland	3.4	0.77	59.3
Taiwan	4.7	0.82	19.9
Greece	3.0	0.75	154
Russia	3.1	0.76	45.1
Serbia (Δe)	3.4	0.77	45.9
Germany (Δe)	6.1	0.86	25.4

Note: Semi-elasticity estimates for Austria, Germany, Hungary and Poland are from Taylor (1991); for Greece and Russia from Engsted (1994). The last two studies are exchange rate models: for Serbia, through June 1993, i.e. short of the last seven months of extreme hyperinflation, Petrovic and Mladenovic (2000), and for Germany, Engsted (1996).

The reported discount factor estimates for individual episodes, as well as its average 0.81, are well below the Engel and West (2005) benchmark of 0.9 (except for Hungary). In general, most semi-elasticity estimates in hyperinflation vary between 3 and 6, giving a range for the discount factor from 0.75 to 0.86. Thus, the result for the Serbian episode seems to generalize for hyperinflation as such.

Nevertheless, when one descends from hyperinflation to high inflation, the value of discount factor seems to decrease and consequently so does the predictive power of the MMER. Comparable estimates of the discount factor for high inflation episodes are reported in Table 7.

Table 7

Semi-elasticity of Money Demand and Discount Factors in High Inflation
Monthly frequency

High-inflation episodes

	Semi-elasticity α	Discount factor $a = \alpha/(1+\alpha)$	Average monthly inflation rate
Chile	16.9	0.94	5.4%
Argentina	12.7	0.93	10.3
Peru	11.8	0.92	6.3
Brazil	11.2	0.92	4.7
Bolivia	7.4	0.88	6.6

Source for the first and the third column: Phylaktis and Taylor (1993)

Estimates of the discount factor are now above the Engel and West (2005) benchmark of 0.9, with average being 0.92, but they are still well below one.

6. *Conclusions*

We found that the out-of-sample forecast of a monetary model of exchange rate clearly outperforms the random walk one in hyperinflation at daily frequency. The result is robust with respect to various out-of-sample test statistics, i.e. CW, Theil-U and DMW, and forecast windows. Nevertheless, it is found that an MMER performs best at the most challenging very short, one-day-ahead forecast, and that its forecasting ability deteriorates as the forecasting horizon increases. This pattern is completely opposite to the standard results found in low-inflation episodes, showing that fundamental-based exchange rate models forecast poorly at one step ahead, only to improve their performance substantially for a several-steps-ahead forecast (cf. Mark, 1995, Engel, Mark and West, 2007).

The found superiority of an MMER forecast in hyperinflation can be attributed to the high speed of adjustment of the exchange rate to its fundamental value and the low, well-below-one discount factor both detected in the Serbian episode as opposed to low-inflation ones. As to the former, the adjustment coefficient is found to be close to 1 for a one-step-ahead forecast, implying almost complete (apart for an impact of the short-run dynamics) adjustment within a period, and, consequently, a declining coefficient as forecasting horizon increases. The completely inverse profile of these coefficients is found in the standard low inflation environment (cf. Mark, 1995). The latter implies a gradual adjustment of the exchange rate to changes in economic fundamentals, and it could be explained by the presence of nominal rigidities in low-inflation episodes (cf. Mark, 1995). However, hyperinflation removes these rigidities, thus suggesting that the fast adjustment detected in the Serbian episode might generalize.

The low discount factor in the present value model of exchange rate found in the Serbian hyperinflation (0.84) places a large weight on the current, as opposed to future, money supply in determining the exchange rate. This implies that the current money supply should be able to predict well the exchange rate, which may explain the good forecasting results of an MMER obtained in this paper. On the contrary, in developed, low-inflation countries the discount factor is just below 1 (cf. Sarno and Sojli, 2009), placing a large weight on the future fundamental in determining the exchange rate, hence resulting in an inferior MMER forecast. Finally, evidence is offered showing that the low discount factor appears in other hyperinflation episodes, implying again that reported findings for one, the Serbian episode, may well generalize.

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Appendix

Respective forecasting errors are the same irrespective of whether one predicts the exchange rate return (Δe_t) or its change ($\Delta^2 e_t$)

A. Random Walk model

1. *Predicting return* Δe

RW model: $\Delta e_t = \Delta e_{t-1} + \varepsilon_t$

Forecast: $\Delta e_{t+1}{}^f = \Delta e_t{}^a$

Forecast error: ε_{t+1}

2. *Predicting first difference of return* Δe^2

RW model: $\Delta^2 e_t = \varepsilon_t$

Forecast: $\Delta^2 e_{t+1} = 0$

Forecast error: ε_{t+1}

B. MMER

1. *Predicting return* Δe

Forecast: $\Delta e_{t+1}{}^f = \Delta e_t{}^a + \gamma(\Delta m - \Delta e)_t$

Forecast error: $\Delta e_{t+1}{}^a - \Delta e_{t+1}{}^f = \Delta e_{t+1}{}^a - \Delta e_t{}^a - \gamma(\Delta m - \Delta e)_t = \varepsilon_{t+1} - \gamma(\Delta m - \Delta e)_t$

2. *Predicting first difference of return* Δe^2

Forecast: $\Delta^2 e_{t+1}{}^f = \gamma(\Delta m - \Delta e)_t$

Forecast error: $\Delta^2 e_{t+1}{}^a - \Delta^2 e_{t+1}{}^f = \varepsilon_{t+1} - \gamma(\Delta m - \Delta e)_t$