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Asymmetric Correlation of Sovereign Bond Yield Dynamics in the Eurozone

Summary: This paper examines the symmetry of correlation of sovereign bond yield dynamics between eight Eurozone countries (Austria, Belgium, France, Germany, Ireland, Italy, Portugal, and Spain) in the period from January 3, 2000 to August 31, 2011. Asymmetry of correlation is investigated pair-wise by applying the test of Yongmiao Hong, Jun Tu, and Guofu Zhou (2007). Whereas the test of Hong, Tu, and Zhou (2007) is static, the present paper provides also a dynamic version of the test and identifies time periods when the correlation of Eurozone sovereign bond yield dynamics became asymmetric. We identified seven pairs of sovereign bond markets for which the null hypothesis of symmetry in correlation of sovereign bond yield dynamics can be rejected. Calculating rolling-window exceedance correlation, we found that the time-varying upper- (i.e. for positive yield changes) and lower-tail correlations (i.e. for negative yield changes) for pair-wise observed sovereign bond markets normally follow each other closely, yet during some time periods (for most pair-wise observed countries, these periods are around the September 11 attack on the New York City WTC and around the start of the Greek debt crisis) the difference in correlation does increase. The results show that the upper- and lower-tail correlation was symmetric before the Eurozone debt crisis for most of the pair-wise observed sovereign bond markets but has become much less symmetric since then.

Key words: Asymmetric correlation, Sovereign bonds, Eurozone, Financial crisis.

JEL: F21, G15, G12, H63.

The study of asymmetric correlations is important from the perspective of optimal portfolio allocation and risk management. Since the seminal works of Harry Markowitz (1952), it is recognized that international diversification reduces a portfolio's total risk. This is due to non-perfect positive comovement between returns of the portfolio assets. Increased comovement between asset returns can then diminish the advantage of diversified investment portfolios (Ling Xiao and Gurjeet Dhesi 2010). The benefits of diversification have further been questioned by a number of studies that document that correlations between asset returns are higher in down markets, when returns fall, than in up markets, when returns rise. Well-diversified portfolio protects investors' wealth during down markets, but the existence of asymmetric correlation (asymmetric dependence) implies that the benefits of diversification can be substantially reduced. As Hong, Tu, and Zhou (2007) show, incorporating the asymmetry in correlations into portfolio decisions can add substantial economic value to investors. The knowledge of whether correlation between assets returns is asymmetric is relevant also for supervisory authorities because of their implications for the stability of the global financial system (Andrew Clare and Ilias Lekkos 2000; Robert-Paul Berben and Jos W. Jansen 2005). Given the need of stronger coordination between monetary and fiscal policy during turbulent financial market conditions (Philip Arestis 2011), the issue is of great importance also for conduct of monetary policy.

This paper provides the most recent evidence on symmetry of correlation of Eurozone sovereign yield dynamics, applying the statistical test of Hong, Tu, and Zhou (2007). The paper consists of four related sections. After the introduction, the first section provides the literature review of studies of asymmetric correlation. The second section presents the methodology of testing the asymmetry of correlation in sovereign bond market yield dynamics. The third section describes the data and analyses the empirical results while the fourth section concludes the paper and presents the final reflections.

1. Literature Review

The empirical literature offers plenty of evidence of asymmetric correlation for different portfolios of stocks or stock market returns (Andrew Ang and Geert Bekaert 2000; François Longin and Bruno Solnik 2001; Ang and Joe Chen 2002; Kee Hong Bae, Andrew G. Karolyi, and René Stulz 2003; Lorenzo Cappiello, Robert F. Engle, and Kevin Sheppard 2003; Hong, Tu, and Zhou 2007; Jamie Alcock and Anthony Hatherley 2009; Byoung Uk Kang et al. 2010; Silvo Dajcman, Mejra Festic, and Alenka Kavkler 2012), yet only a few studies have investigated asymmetric correlation in bond market returns or yields (Cappiello, Engle, and Sheppard 2003; Jian Yang, Yinggang Zhou, and Kin W. Leung 2012).

Assessing asymmetric correlation requires care. Brian H. Boyer, Michael S. Gibson, and Mico Loretan (1999) and Kristin Forbes and Roberto Rigobon (2002) find that a correlation computed as conditional on some variables being high or low is a biased estimator of the unconditional correlation. Even if one obtains from the real data that a conditional correlation is much higher than the unconditional sample correlation, it is not sufficient to claim the existence of asymmetric correlations (Hong, Tu, and Zhou 2007).

The most common methods of analyzing time-varying and potentially asymmetric dependence structures include the multivariate GARCH models (Cappiello, Engle, and Sheppard 2003; Yang, Zhou, and Leung 2012), copula functions (Kang et al. 2010) and the recently developed tests of asymmetric dependence (Longin and Solnik 2001; Ang and Chen 2002; Hong, Tu, and Zhou 2007). Hong, Tu, and Zhou (2007) provide a statistically robust test of asymmetric dependence between time series. The test has three appealing features: it is a model-free test, so it can be used without having to specify a statistical model for the data; unlike many of asymmetry tests that impose normality assumption on the data, the test allows for general distributional assumptions, such as the GARCH process; the test statistic is easy to implement; and its asymptotic distribution follows a standard chi-square distribution under the null hypothesis of symmetry. The test is based on a measure of exceedance correlation that circumvents problems associated with the correlation coefficient. This is because co-exceedances are not biased in periods of high volatility and are not restricted to modeling linear phenomena like the measure of linear (Pearson's) correlation is (see Dirk Baur and Niels Schulze 2005; Mardi Dungey et al. 2005).

In this paper we apply the test of Hong, Tu, and Zhou (2007) and investigate asymmetry of correlation between Eurozone sovereign yield dynamics, by estimating exceedance correlation between yield dynamics of two sovereign bond markets at a time. Following Longin and Solnik (2001), Ang and Chen (2002), and Hong, Tu, and Zhou (2007), exceedance is defined as an occurrence of a change in a yields of sovereign bonds of a particular country that is above or below a certain exceedance level. Whereas Ang and Chen (2002) and Hong, Tu, and Zhou (2007) define exceedance level in terms of standard deviations, this paper defines exceedance in terms of the percentiles of the empirical cumulative distribution function of the yield changes. The test of Hong, Tu, and Zhou (2007) is static, assessing symmetry of correlation for a whole observed period. In the present paper, we also provide a dynamic version of the test of correlation symmetry of Hong, Tu, and Zhou (2007) and examine time periods during which the correlation of Eurozone sovereign bond yield dynamics became asymmetric.

2. Methodology

Asymmetry in correlation of bond yield dynamics is investigated pair-wise by estimating the exceedance correlation between bond yield changes (bond yield dynamics) of two sovereign bond markets at a time. Following Longin and Solnik (2001), Ang and Chen (2002), and Hong, Tu, and Zhou (2007), exceedance is defined as an occurrence of a change in yields of sovereign bonds of a particular country that is above or below a certain exceedance level. Whereas Ang and Chen (2002) and Hong, Tu, and Zhou (2007) define exceedance level in terms of standard deviations, this paper defines exceedance in terms of percentiles of the empirical cumulative distribution function of the yield changes. More specifically, let $\{y_{1t}, y_{2t}\}$ be the changes in yields of two sovereign bond markets (i.e. sovereign bonds of two different countries) in period t. A correlation at an exceedance level c is defined as correlation between the two time series when both of them are at least c percentile away from their median yield change in the empirical distribution function:

$$\rho^{+}(c) = corr(y_{1t}, y_{2t} | y_{1t} > c, y_{2t} > c), \tag{1a}$$

$$\rho^{-}(c) = corr(y_{1t}, y_{2t} | y_{1t} < -c, y_{2t} < -c), \tag{1b}$$

where the yield changes are standardized to have zero median and unit variance so that the median and variance do not appear explicitly in the right-hand side of Equations (1a) and (1b).

Following Longin and Solnik (2001), Ang and Chen (2002), and Hong, Tu, and Zhou (2007) we test whether correlation between the upper- and the lower-tail of yield changes of the two sovereign bond markets is the same. Thus, the null hypothesis of symmetric correlation is

$$H_0: \rho^{+}(c) = \rho^{-}(c), \text{ for all } c \in (0{,}0.5). \tag{2}$$

Hong, Tu, and Zhou (2007) provide a model-free test for the null of symmetric exceedance correlations. They show that for the m chosen number of different exceedance levels, the vector:

$$\hat{\rho}^{+}(c)-\hat{\rho}^{-}(c)=[\hat{\rho}^{+}(c_1)-\hat{\rho}^{-}(c_1),\ldots,\hat{\rho}^{+}(c_m)-\hat{\rho}^{-}(c_m)]', \tag{3}$$

after being scaled by $\sqrt{T}$ (T is the size of a random sample of the yield changes series), under the null hypothesis of symmetry has an asymptotic normal distribution with zero mean and positive definite variance-covariance matrix, Ω, for all possible true distributions of the data.

Under condition that y_{1t} and y_{2t} are simultaneously in the upper c percentile tail, the sample means and variance of these conditional time series are computed:

$$\hat{\mu}_1^{+}(c)=\frac{1}{T_c^{+}}\sum_{t=1}^{T} r_{1t}|(y_{1t}>c, y_{2t}>c), \tag{4a}$$

$$\hat{\mu}_2^{+}(c)=\frac{1}{T_c^{+}}\sum_{t=1}^{T} r_{2t}|(y_{1t}>c, y_{2t}>c), \tag{4b}$$

$$\hat{\sigma}_1^{+}(c)^2=\frac{1}{T_c^{+}-1}\sum_{t=1}^{T}[y_{1t}-\hat{\mu}_1^{+}(c)]^2|(y_{1t}>c, y_{2t}>c), \tag{4c}$$

$$\hat{\sigma}_2^{+}(c)^2=\frac{1}{T_c^{+}-1}\sum_{t=1}^{T}[y_{2t}-\hat{\mu}_2^{+}(c)]^2|(y_{1t}>c, y_{2t}>c), \tag{4d}$$

where $\hat{\mu}_1^{+}(c)$ and $\hat{\mu}_2^{+}(c)$ are the estimated conditional means of the series and $\hat{\sigma}_1^{+}(c)^2$ and $\hat{\sigma}_2^{+}(c)^2$ the estimated conditional variance of the series.

The sample conditional correlation $\hat{\rho}^{+}(c)$ is then given by:

$$\hat{\rho}^{+}(c)=\frac{1}{T_c^{+}-1}\sum_{t=1}^{T}\hat{X}_{1t}^{+}(c)\hat{X}_{2t}^{+}(c)\big|(y_{1t}>c, y_{2t}>c), \tag{5}$$

where $\hat{X}_{1t}^{+}(c)=\frac{y_{1t}-\hat{\mu}_1^{+}(c)}{\hat{\sigma}_1^{+}(c)}$, and $\hat{X}_{2t}^{+}(c)=\frac{y_{2t}-\hat{\mu}_2^{+}(c)}{\hat{\sigma}_2^{+}(c)}$. The same computations apply also for $\hat{\rho}^{-}(c)$.

Hong, Tu, and Zhou (2007) prove that the null hypothesis of symmetric correlation can be tested by the J_ρ statistics which under the null hypothesis and under certain regularity conditions is asymptotically chi-square distributed with m degrees of freedom

$$J_\rho=T(\hat{\rho}^{+}(c)-\hat{\rho}^{-}(c))'\hat{\Omega}^{-1}(\hat{\rho}^{+}(c)-\hat{\rho}^{-}(c))\sim\chi_m^2, \tag{6}$$

where $\hat{\rho}^{+}(c)-\hat{\rho}^{-}(c)$ is defined by Equation (3) and $\hat{\Omega}$ is consistent estimate of the asymptotic covariance matrix of $\hat{\rho}^{+}(c)-\hat{\rho}^{-}(c)$. The variance-covariance matrix is given by:

$$\hat{\Omega}=\sum_{l=1-T}^{T-1} k\left(\frac{l}{p}\right)\hat{\gamma}_l, \tag{7}$$

where $\hat{\gamma}_l$ is an $N\times N$ matrix with (i,j)-th element

$$\hat{\gamma}_l(c_i,c_j)=\frac{1}{T}\sum_{t=|l|+1}^{T}\hat{\zeta}_t(c_i)\,\hat{\zeta}_{t-|l|}(c_j), \tag{8}$$

and

$$\hat{\zeta}_t(c) = \frac{T}{T_c^+}\left[\hat{X}_{1t}^+(c)\hat{X}_{2t}^+(c) - \hat{\rho}^+(c)\right]\Big|(y_{1t} > c, y_{2t} > c) - \\ -\frac{T}{T_c^-}\left[\hat{X}_{1t}^-(c)\hat{X}_{2t}^-(c) - \hat{\rho}^-(c)\right]\Big|(y_{1t} < -c, y_{2t} < -c) \quad (9)$$

where $k(\cdot)$ is a Bartlett kernel function that that assigns a suitable weight to each lag of order l and p is the smoothing parameter or lag truncation order.

$\hat{\rho}^+(c)$ and $\hat{\rho}^-(c)$ as proposed by Hong, Tu, and Zhou (2007) are estimated statically, obtaining one measure of correlation for the whole sample. In this paper we also estimate $\hat{\rho}^+(c)$ and $\hat{\rho}^-(c)$ dynamically by calculating rolling-window correlations. Using this approach, correlations $\hat{\rho}^+(c)$ and $\hat{\rho}^-(c)$ between two time series of yield changes at time t are computed from w observations (where w is the size of the window), centered around time t. The window is rolled forward one day at a time, resulting in a time series of exceedance correlations. This way we obtain $T - w$ correlation coefficients.

3. Data and Empirical Results

We investigate symmetry of exceedance correlations of sovereign bond yield changes for the Eurozone countries listed in Table 1. The daily bond yield changes were calculated from the yields (y) of central-government bonds (bullet issues) with 10 years maturity as $ln(y_t) - ln\ (y_{t-1})$ (bond yield changes are calculated the same way as in both Alain Durré and Pierre Giot 2005 and Sangbae Kim and Francis In 2007). Days with no trading in any of the observed markets were left out. The data for bond yields are from Denmark's Central bank. Table 1 presents some descriptive statistics of the data.

Table 1 Descriptive Statistics of Bond Yield Changes

	Period of observation	Min	Max	Mean	Std. deviation	Skewness	Kurtosis	Jarque-Bera statistics
Austria	3 January 2000 – 31 August 2011	-0.06209	0.05129	-0.000229	0.01044	0.1528	5.0936	554.89***
Belgium	3 January 2000 – 31 August 2011	-0.04134	0.06603	-0.000117	0.01023	0.3367	5.6287	912.79***
France	3 January 2000 – 31 August 2011	-0.04921	0.06003	-0.000220	0.01059	0.1360	4.7921	407.3***
Ireland	3 January 2000 – 31 August 2011	-0.215	0.08457	0.000139	0.01237	-1.3056	38.1730	15,419.9***
Italy	3 January 2000 – 31 August 2011	-0.1406	0.07526	-0.000036	0.00992	-0.6834	19.7355	34,949.3***
Germany	3 January 2000 – 31 August 2011	-0.07596	0.07637	-0.000303	0.01208	0.0345	6.3872	1,422.8***
Portugal	3 January 2000 – 31 August 2011	-0.3006	0.1449	0.0002257	0.01358	-3.3664	93.2459	1.015,175.3***
Spain	3 January 2000 – 31 August 2011	-0.1582	0.06068	-0.000039	0.01101	-1.2329	23.6001	53,357.3***

Note: Jarque-Bera statistics: *** indicate that the null hypothesis of normal distribution is rejected at a 1% significance level.

Source: Own calculations.

All series display significant leptokurtic behavior as evidenced by large kurtosis with respect to the Gaussian distribution. The Jarque-Bera test rejects the hypothesis of a normally distributed observed time series. Also, the stationarity of bond yield changes was checked by the Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test. The rejection of the null hypothesis of ADF and PP tests and non-rejection of the null hypothesis of KPSS test led to the conclusion of no unit-root in the time series (the results are not reported here, but can be obtained from the author).

As we study the asymmetric correlation at the exceedance levels defined in terms of percentiles of the empirical cumulative distribution function of the bond yield changes, we next report (Table 2) the 5th, 15th, 30th, 50th, 75th, and the 95th percentiles of sovereign bond yield changes.

The median change of sovereign bond yield series for all the investigated countries' time series is zero. $\rho^{+}(c)$ is thus a correlation between positive values of y_{1t} and y_{2t} (i.e. correlation in the case of increasing yields or correlation in a down market), while $\rho^{-}(c)$ is the correlation between negative values of y_{1t} and y_{2t} (correlation in up market).

Table 2 The Percentiles of Daily Sovereign Bond Yield Changes

	Percentile of daily bond yield changes					
	5th	15th	30th	50th	75th	95th
Austria	-0.0170	-0.0097	-0.0049	0	0.0055	0.0169
Belgium	-0.0164	-0.0095	-0.0048	0	0.0056	0.0166
France	-0.0173	-0.0100	-0.0052	0	0.0058	0.0169
Ireland	-0.0168	-0.0097	-0.0046	0	0.0058	0.0182
Italy	-0.0146	-0.0087	-0.0043	0	0.0054	0.0154
Germany	-0.0183	-0.0107	-0.0054	0	0.0058	0.0188
Portugal	-0.0169	-0.0094	-0.0043	0	0.0058	0.0195
Spain	-0.0167	-0.0094	-0.0048	0	0.0057	0.0172

Source: Own calculations.

Table 3 reports Pearson's correlation coefficients between the logarithmic bond yield changes. Pearson's correlation is a linear measure of comovement, calculated across the whole spectrum of empirical yield changes; i.e. for all observation of y_t, where an equal weight to each observation is given. Notably, the greatest correlation of bond yield changes in the observed period was achieved between the sovereign bond pairs of France-Germany, Austria-France, and Italy-Spain, while comovement between the yield changes of the sovereign bonds of Germany-Portugal was the smallest of all.

Table 3 Pearson's Correlation between Sovereign Bond Yield Changes

	Austria	Belgium	France	Germany	Ireland	Italy	Portugal	Spain
Austria	1							
Belgium	0.8577	1						
France	0.9206	0.8663	1					
Germany	0.8671	0.7446	0.9214	1				
Ireland	0.5374	0.6557	0.5277	0.3906	1			
Italy	0.6821	0.8270	0.6856	0.5334	0.7063	1		
Portugal	0.4727	0.6125	0.4690	0.3288	0.8089	0.6854	1	
Spain	0.6642	0.8146	0.6641	0.5286	0.7433	0.9048	0.7299	1

Note: All the correlation coefficients are significantly different from zero.

Source: Own calculations.

Tables 4a and 4b provide the results of the correlation symmetry test for all pair-wise investigated sovereign bond markets. The second column reports the p-value of J_ρ statistics and the third column reports the difference between the correlation of yield dynamics that are below the median yield change (lower-tail dynamics or negative yield changes), $\hat{\rho}^{-}(c)$, and the yield dynamics that are above the median yield change (upper-tail dynamics or positive yield changes), $\hat{\rho}^{+}(c)$. Following Hong, Tu, and Zhou (2007), the test of asymmetric correlation is performed also for a set of exceedance levels, $c = \{0.45, 0.35, 0.2, 0\}$.

For the singleton exceedance level $c = \{0\}$ we can reject the hypothesis of symmetric correlation (at the 5% significance level) of sovereign bond yield changes for the bond market pairs of France-Italy, France-Spain, Ireland-Spain, and Portugal-Spain, but not for the others. For a set of four exceedance levels, $c = \{0.05, 0.15, 0.3, 0.5\}$, the rejection of the null of symmetric correlation in sovereign bond yield dynamics of Austria-Italy, Belgium-Portugal, and Germany-Portugal can now also be rejected, while for the bond markets of Ireland-Spain and Portugal-Spain the null hypothesis of symmetric correlation can no more be rejected.

Comparing the results in Tables 3 and 4a/4b, the difference $\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$ for the markets that are strongly correlated as measured by Pearson's correlation is small compared to the markets that are not so strongly linearly correlated (for instance, Austria-Portugal and Germany-Portugal). A large difference in the upper- and the lower-tail correlation of yield dynamics does not mean that there is necessarily a genuine difference in the population parameters (e.g. the sovereign bond markets of Austria-Spain). As noted by Hong, Tu, and Zhou (2007), there are always differences in the sample estimates due to sample variations.

Table 4a Results of the Test of Correlation Symmetry

Sovereign bond markets	$c = \{0\}$		$c = \{0.45,0.35,0.2,0\}$				
	p-value	$\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$	p-value	$\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$; $c = 0.45$	$\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$; $c = 0.35$	$\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$; $c = 0.2$	$\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$; $c = 0$
Aut-Bel	0.6332	-0.0398	0.9922	-0.0294	-0.0505	-0.0524	-0.0398
Aut-Fra	0.8803	-0.0129	0.9489	0.0386	-0.0232	-0.0071	-0.0129
Aut-Ger	0.8848	-0.0138	0.9954	-0.0053	-0.0100	-0.0055	-0.0138
Aut-Ire	0.5266	-0.0428	0.4025	-0.0341	-0.0382	-0.1009	-0.0428
Aut-Ita	0.5387	0.0589	0.0057	0.1808	0.3128	0.1042	0.0589
Aut-Por	0.4803	-0.0465	0.2408	0.3837	0.1394	0.0122	-0.0465
Aut-Spa	0.4276	0.0724	0.7921	0.2338	0.1969	0.1246	0.0724
Bel-Fra	0.9036	-0.0104	0.9893	-0.0559	-0.0096	-0.0001	-0.0104
Bel-Ger	0.9808	0.0022	0.5366	-0.0030	0.0094	-0.0365	0.0022
Bel-Ire	0.4822	0.0584	0.7425	0.1246	0.1064	0.0538	0.0584
Bel-Ita	0.5475	0.0637	0.6935	0.2038	0.1575	0.1290	0.0637
Bel-Por	0.4512	0.0648	0.0196	0.2311	0.2096	0.0429	0.0648
Bel-Spa	0.3395	0.0955	0.9019	0.2169	0.1774	0.1358	0.0955

Notes: Two sets of exceedance levels were used to perform the test. The first is the singleton $c = \{0\}$ and the second is $c = \{0.45,0.35,0.2,0\}$. $\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$ represents the difference in correlation between two time series of bond yield changes that are simultaneously either at least percentiles above or below the median yield change of the time series. p-value is a significance level of rejecting the null hypothesis of symmetric correlation.

Source: Own calculations.

Table 4b Results of the Test of Correlation Symmetry

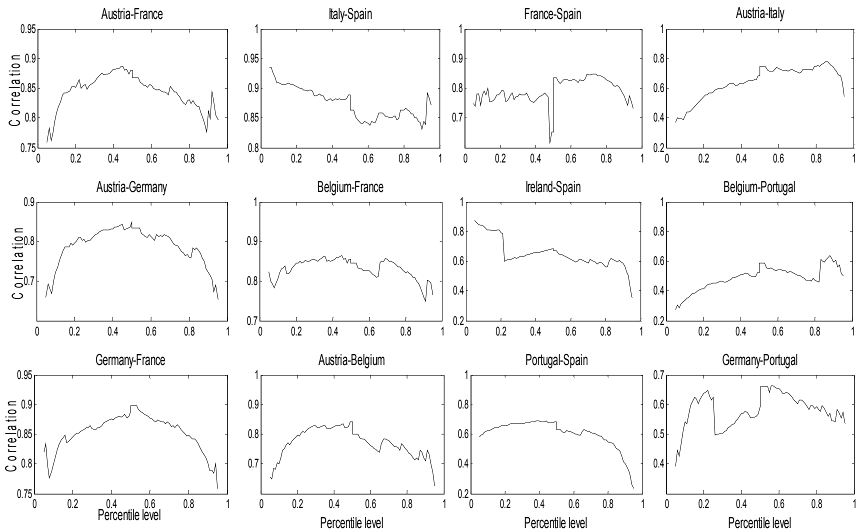
Sovereign bond markets	$c = \{0\}$		$c = \{0.45, 0.35, 0.2, 0\}$				
	p-value	$\hat{\rho}^+(c) - \hat{\rho}^-(c)$	p-value	$\hat{\rho}^+(c) - \hat{\rho}^-(c)$; $c = 0.45$	$\hat{\rho}^+(c) - \hat{\rho}^-(c)$; $c = 0.35$	$\hat{\rho}^+(c) - \hat{\rho}^-(c)$; $c = 0.2$	$\hat{\rho}^+(c) - \hat{\rho}^-(c)$; $c = 0$
Fra-Ger	0.9182	0.0100	0.9964	-0.0614	-0.0097	0.0084	0.0100
Fra-Ire	0.7728	-0.0209	0.6747	-0.1529	-0.1315	-0.0338	-0.0209
Fra-Ita	0.0181	0.1820	0.0003	0.0224	0.1090	0.0709	0.1820
Fra-Por	0.6243	0.0346	0.7529	-0.1293	0.0489	0.0320	0.0346
Fra-Spa	0.0169	0.1821	0.0005	-0.0156	0.0498	0.0716	0.1821
Ger-Ire	0.8856	-0.0114	0.9335	0.1095	-0.0185	-0.0050	-0.0114
Ger-Ita	0.2780	0.1045	0.3596	0.3089	0.3102	0.1701	0.1045
Ger-Por	0.4215	0.0624	0.0507	0.1462	-0.0477	0.0977	0.0624
Ger-Spa	0.2603	0.1054	0.1921	0.3331	0.3234	0.1863	0.1054
Ire-Ita	0.9329	-0.0127	0.3526	-0.5476	-0.2292	-0.0420	-0.0127
Ire-Por	0.7761	-0.1109	0.9892	-0.3086	-0.2046	-0.1584	-0.1109
Ire-Spa	0.0056	-0.0158	0.3320	-0.5171	-0.1945	-0.0391	-0.0158
Ita-Por	0.8751	-0.0271	0.5101	-0.0590	0.0426	-0.0083	-0.0271
Ita-Spa	0.9183	-0.0264	0.9997	-0.0629	-0.0517	-0.0351	-0.0264
Por-Spa	0.0470	-0.0519	0.2477	-0.3449	-0.1414	-0.0552	-0.0519

Notes: See notes for Table 4a.

Source: Own calculations.

Figure 1 presents the plots of correlation coefficients $\hat{\rho}^+(c)$ and $\hat{\rho}^-(c)$ for a set of 46 exceedance levels $c = \{0.01, 0.02, \dots, 0.06, \dots, 0.45\}$). The first two column plots are for the sovereign bond markets with the highest Pearson's correlation and for which the symmetry in correlations is not rejected by the results of Tables 4a and 4b. The last two columns of the plots represent the markets for which the results in Tables 4a and 4b led to rejection of the hypothesis of symmetric correlation. Notably, for sovereign bond markets of Austria-France, Austria-Germany, Germany-France, Belgium-France, and Austria-Belgium the correlation curve is symmetric and inversely U-shaped. The correlation between these pair-wise markets is thus more strong at smaller (positive and negative) yield changes and reduces as the yield changes get larger. At the 5th and 95th percentile of the empirical distribution of yield changes correlation coefficients are very similar in absolute value. Correlation between extreme positive and extreme negative yield changes is thus still symmetric.

For the sovereign bond markets of Ireland-Spain, Portugal-Spain, and other pairs of the markets for which the plots are drawn in the last two columns, such symmetry cannot be observed. For the Ireland-Spain and Portugal-Spain sovereign bond markets, we found that lower-tail yield dynamics in these pair-wise observed markets are more strongly correlated than upper-tail yield dynamics. The opposite holds true for the sovereign bond markets of Austria-Italy, Belgium-Portugal, and Germany-Portugal; these markets comove more strongly when yield changes are above the median than when they are below the median. The results of the symmetry correlation test in Table 4b show that the comovement between sovereign bond markets of Spain and France is asymmetric. Yet, Figure 1 reveals that this is largely the result of asymmetric comovement when deviations in the yield dynamics from the median yield dynamic are small, whereas for larger deviations the correlation is more symmetric.



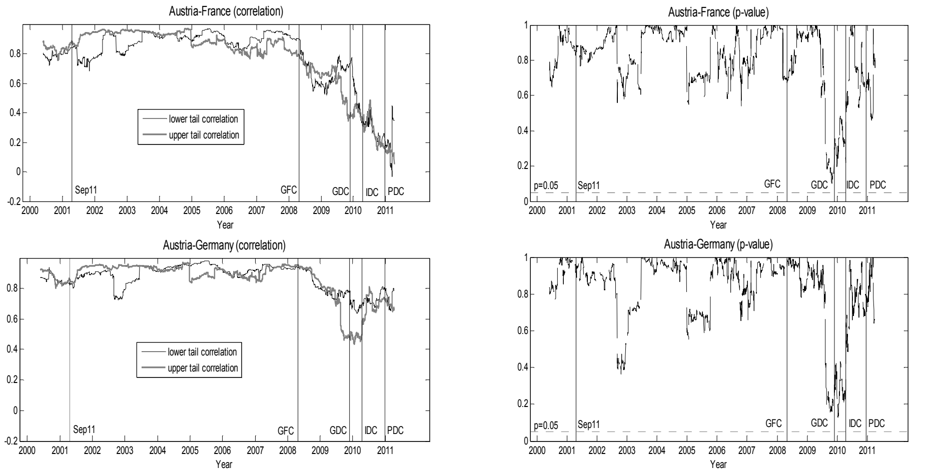
Notes: The plots show the correlation between sovereign bond yields at diverse percentile levels (from the 5th (level 0.05) percentile up to 95th (level 0.95) percentile) of the empirical cumulative distribution function of the sovereign yield changes. The first two column plots are for the sovereign bond markets with the highest Pearson's correlation, and the last two columns of plots are for the markets with significant asymmetric correlation.

Source: Own calculations.

Figure 1 Correlation between Yield Changes at Diverse Percentile Levels of Empirical Cumulative Distribution Function of Yield Changes for Particular Sovereign Bond Markets

The upper-tail correlation coefficients, $\hat{\rho}^{+}(c)$, the lower-tail correlation coefficients, $\hat{\rho}^{-}(c)$, and the tests of symmetry (i.e. the results presented in Tables 3a and 3b, and Figure 1) are computed for the whole observation period (from January 3, 2000 to August 31, 2011) and are thus static estimates. To examine how the symmetry of correlation has changed through time, we need to compute time-varying $\hat{\rho}^{+}(c)$ and $\hat{\rho}^{-}(c)$. These are computed as rolling-window correlations of $\hat{\rho}^{+}(c)$ and $\hat{\rho}^{-}(c)$(between two time series of yield changes) at time t from w observations (where w is the size of the window), centered around time t. The window is rolled forward one day at a time, resulting in a time series of correlations. This way we obtain $T-w$ correlation coefficients. The window size has to capture enough data points to obtain reasonable estimates for higher scales. We take $c=0$ and thus compute just the correlation for the positive and correlation for the negative yield changes for the pair-wise investigated sovereign bond markets. Next, we choose $w=200$ days, so we have enough observations to calculate correlation for the positive and the negative yield changes. Rolling-window J_{ρ} and p-values are computed too, yet only rolling p-values are graphically presented.

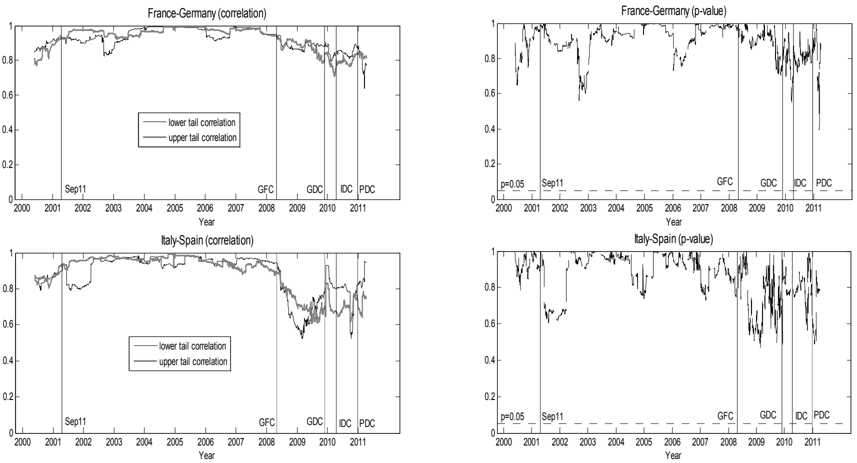
Figures 2a and 2b presents rolling window correlations for the positive (down market) and the negative (up market) yield changes (thus c is set to zero) for the country pairs listed in the first two columns in Figure 1, while Figure 3 show the same for the countries listed in the last two columns in Figure 1.



Notes: Plotted are the rolling window correlations for the positive (upper-tail correlations; the down market) and the negative (lower-tail correlations; the up market) yield changes (thus is set to zero).The left-side plots present the time-varying upper- and lower-tail correlation estimates, while the right-side plots the time-varying significance levels (p-value) of rejecting the null hypothesis of symmetric correlation. On the time axis, the financial turmoil events are denoted: Sep11 = the September 11 attack on the WTC (September 11, 2001), GFC = the global financial crisis (September 16, 2008, the collapse of Lehman Brothers, is denoted), GDC = the Greek debt crisis (April 23, 2010, when the Greek government requested a bailout from the EU/IMF is denoted), IDC = Ireland's debt crisis (September 1, 2010, when the Irish government started negotiations for a bailout with the ECB/IMF is denoted), PDC = Portugal's debt crisis (May 16, 2011 is denoted, when Eurozone leaders approved a bailout of financial help for Portugal). The vertical lines, denoting financial turmoil events, are drawn 100 days (half the window length) before the actual date of the event, as due to the construction characteristics of the moving-window correlation coefficients, the potential effect of the event should start to show up in the graph 100 days before the actual time of the event.

Source: Own calculations.

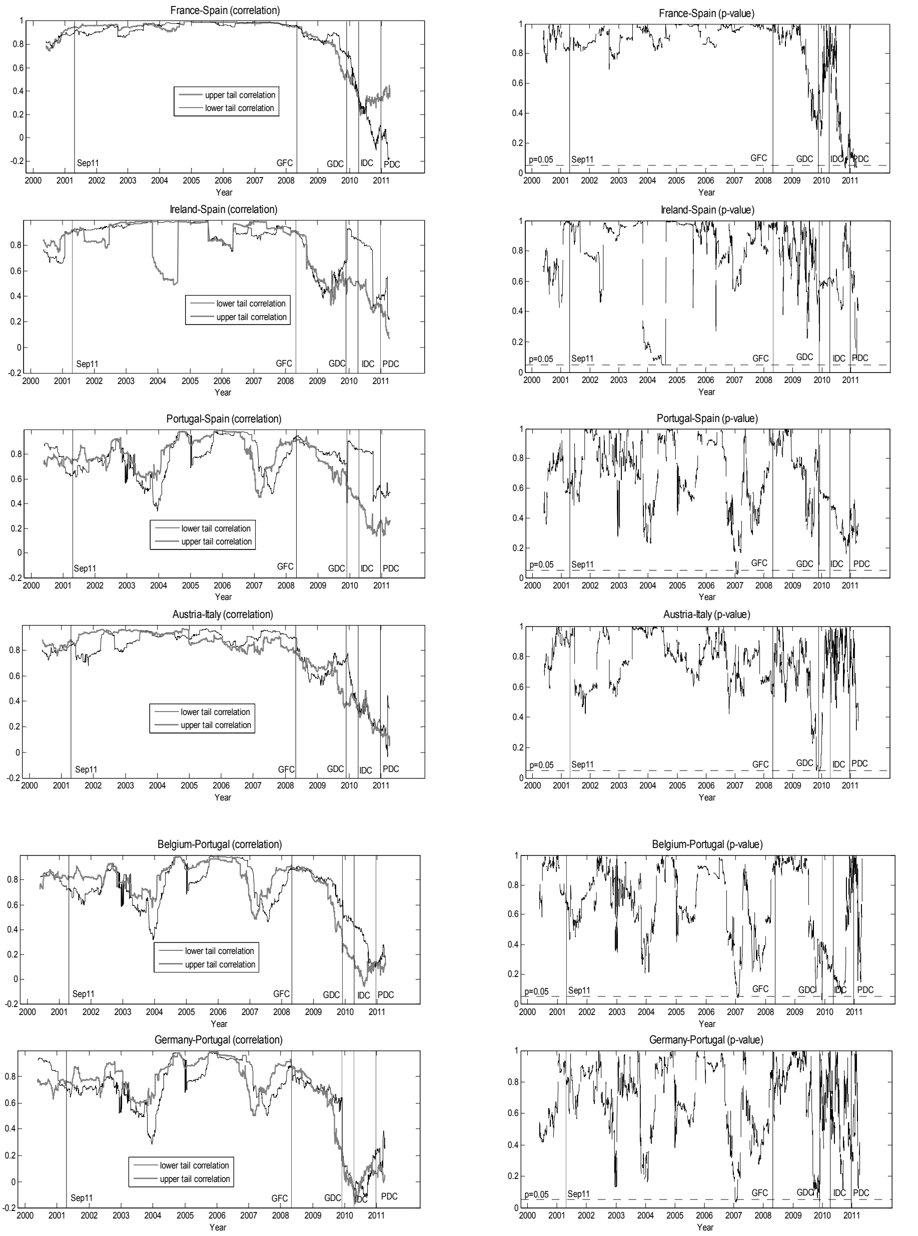
Figure 2a Dynamic Estimates of Upper- (Increasing Yields) and Lower-tail (Decreasing Yields) Correlations and the p-value of the Test of Correlation Symmetry



Notes: See notes for Figure 2a.

Source: Own calculations.

Figure 2b Dynamic Estimates of Upper- (Increasing Yields) and Lower-tail (Decreasing Yields) Correlations and the p-value of the Test of Correlation Symmetry



Notes: See notes for Figure 2a.

Source: Own calculations.

Figure 3 Dynamic Estimates of Upper- (Increasing Yields) and Lower-tail (Decreasing Yields) Correlations and the p-value of the Test of Correlation Symmetry

Evidently, correlations are time varying. Figures reveal that time-varying upper- and lower-tail correlations normally follow each other closely, yet during some time periods (for most pair-wise observed countries, this is around the September 11 attack on New York WTC and around the start of the Greek debt crisis), the difference $\hat{\rho}^{+}(c) - \hat{\rho}^{-}(c)$ does increase. For all pair-wise observed sovereign bond markets, as drawn in Figures 2a, 2b and 3, it is evident that from the start of the global financial crisis the correlation of sovereign bond yield dynamics had reduced, as the financial markets already started to question the sustainability of public debt in some Eurozone countries. Yet, it was not before it became evident that Greece needed a bailout (see Georgios P. Kouretas and Prodromos Vlamis 2010 for a review on causes and consequences of Greece's debt crisis) that the difference between the upper- and lower-tail correlations of sovereign bond yield dynamics has widened across the Eurozone sovereign bond markets.

Notably, correlation between the sovereign bond yield dynamics of Germany and France has not only been the highest of all pair-wise analyzed sovereign bond markets on average, but also the most stable throughout the observed period. The correlation has not reduced much, either, after the start of the Eurozone debt crisis. This cannot be observed for some other pairs of sovereign bond markets, though. Correlation between sovereign bond markets that were not (so much) affected by the rising yields (in Figures 2a, 2b and 3 these are Germany, France, Austria, and to some extent Belgium) and those that were (more) affected (Greece, Ireland, Spain, and Italy) has reduced by more than a half; for some sovereign bond yield dynamics pairs the correlation even turned negative.

The estimated J_{ρ} statistics and the p-values for testing the null hypothesis of symmetric correlation show that correlation in up and down market was symmetric before the Eurozone debt crisis for most of the pair-wise observed sovereign bond markets (except for Ireland-Spain, Portugal-Spain, Belgium-Portugal, and Germany-Portugal), but has become much less symmetric since then. In fact, the results of the test of symmetric correlation prove that correlation of sovereign bond yield dynamics of Belgium-France, France-Spain, Portugal-Spain, Austria-Italy, and Germany-Portugal became asymmetric after the start of Eurozone debt crisis, at least for a short time period.

The results of the paper have important risk management and diversification effect implications for investors in the observed sovereign bond markets. Firstly, as evidenced by the results of this paper, bond yield dynamics can be more interdependent in the extreme market environment, a phenomenon which cannot be captured by a simple (Pearson's) correlation. In order to achieve the best possible diversification effects, the dependence analysis that the investors perform during the portfolio selection analysis, and when rebalancing their portfolio, should consider measures of dependence that are capable of capturing nonlinear dependence structure. Secondly, the investors must consider that the dynamics of sovereign bond yield dynamics dependence structure is time-varying, thus implying the need for portfolio rebalancing. Thirdly, the finding of asymmetric correlation (asymmetric dependence) implies that the benefits of diversification can be substantially reduced, especially in down markets. Incorporating the asymmetry in correlations into portfolio decisions therefore can add substantial economic value to investors.

4. Conclusion

In this paper we investigated the symmetry of correlation of sovereign bond yield dynamics of eight Eurozone countries (Austria, Belgium, France, Germany, Ireland, Italy, Portugal, and Spain) in the period from January 3, 2000 to August 31, 2011. Asymmetry in correlation was investigated pair-wise, by estimating the exceedance correlation between sovereign bond yield changes of two sovereign bond markets at a time and applying the test of symmetry in correlation of Hong, Tu, and Zhou (2007). The results show that for seven pairs of sovereign bond markets, the hypothesis of symmetry in correlation of sovereign bond yield dynamics can be rejected.

Whereas the test of Hong, Tu, and Zhou (2007) is static and tests the symmetry of correlation for a whole observed period only, the present paper provides also a dynamic version of the test and identifies time periods when the correlation between the Eurozone sovereign bond yield dynamics became asymmetric. The results of the dynamic version of the symmetry test of Hong, Tu, and Zhou (2007) show that the correlation in up and down markets was symmetric before the Eurozone debt crisis for most of the pair-wise observed sovereign bond markets (except for Ireland-Spain, Portugal-Spain, Belgium-Portugal, and Germany-Portugal), but has become much less symmetric since then. In fact, the results of the test of symmetric correlation prove that correlation between the positive and the negative yield dynamics between sovereign bonds of Belgium-France, France-Spain, Portugal-Spain, Austria-Italy, and Germany-Portugal became asymmetric after the start of the Eurozone debt crisis, at least for a short time.

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